

**Exercise 36**

Find the derivative. Simplify where possible.

$$y = \operatorname{sech} x(1 + \ln \operatorname{sech} x)$$

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**Solution**

Take the derivative using the chain and product rules.

$$\begin{aligned} y' &= \frac{d}{dx}[\operatorname{sech} x(1 + \ln \operatorname{sech} x)] \\ &= \left[ \frac{d}{dx}(\operatorname{sech} x) \right] (1 + \ln \operatorname{sech} x) + \operatorname{sech} x \left[ \frac{d}{dx}(1 + \ln \operatorname{sech} x) \right] \\ &= (-\operatorname{sech} x \tanh x)(1 + \ln \operatorname{sech} x) + \operatorname{sech} x \left[ \frac{1}{\operatorname{sech} x} \cdot \frac{d}{dx}(\operatorname{sech} x) \right] \\ &= (-\operatorname{sech} x \tanh x)(1 + \ln \operatorname{sech} x) + \operatorname{sech} x \left[ \frac{1}{\operatorname{sech} x} \cdot (-\operatorname{sech} x \tanh x) \right] \\ &= (-\operatorname{sech} x \tanh x)(1 + \ln \operatorname{sech} x) + (-\operatorname{sech} x \tanh x) \\ &= -\operatorname{sech} x \tanh x[(1 + \ln \operatorname{sech} x) + 1] \\ &= -\operatorname{sech} x \tanh x(2 + \ln \operatorname{sech} x) \end{aligned}$$